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ORTHOGONAL DERIVATIONS AND BIDERIVATIONS ON SEMIPRIME SEMIRING

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ABSTRACT

Motivated by some results on Orthogonal Derivations and Biderivations, in [5], the authors defined the notion of orthogonality between the Derivation and Biderivation in ring and some interesting results.In this paper we also introduce the notion of Derivations and Biderivations are orthogonal in semiprime semiring and giving some necessary and sufficient conditions for Derivations and Biderivations are Orthogonal..

Keywords: *Derivation, Bi derivations, Orthogonality.*

I. INTRODUCTION

In [3] Bresar and Vukman initiated the notion of orthogonality for two derivations on a semiprime ring, and they presented several necessary and sufficient conditions for two derivations to be orthogonal are obtained. They also obtained a counter part of a result of Posner. In [5] Daif.M.N, Tammam El-Sayiad and Haetinger.C, introduce the notion of Orthogonality for Derivations and Biderivations. Chandramouleeswaran and Tiruveni in [6], they are first introduce the notion of Derivations on Semiring. These are the motivated for us. So, we introduce the notion of Orthogonality between the Derivations and Biderivations. The main theorem of this paper gives the four conditions are equivalent for the notion of Orthogonality.

II. PRELIMINARIES

Definition: 2.1

A **Semiring** (S,+,•) is a non-empty set S together with two binary operations, + and $\cdot$ such that
(1). (S,+) and (S,•) are a monoid with identity 0 and 1.
(2). For all a, b, c ϵ S, $a \,.\, (b + c) = a \,.\, b + a \,.\, c$ and $(b + c) \,.\, a = b \,.\, a + c \,.\, a$

Definition: 2.2

A semiring S is said to be **2- torsion free** if $2x = 0 \Rightarrow x = 0, \forall x \in S$.

Definition: 2.3

A semiring S is **Prime** if $xSy = 0 \Rightarrow x = 0$ or $y = 0$, $\forall\, x, y \in S$ and S is **Semi Prime** if $x S x = 0 \Rightarrow x = 0, \forall\, x \in S$.

Definition: 2.4

An additive map $d : S \rightarrow S$ is called a **derivation** if $d(xy) = d(x)\, y + x\, d(y), \forall\, x, y \in S$

Definition: 2.5

A biadditive mapping $D: S \times S \rightarrow S$ is called a **biderivation** if $D(xy,z) = D(x,z)\, y + x\, D(y,z), \forall\, x, y, z \in S$. Obviously, in this case also the relation $D(x,yz) = D(x,y)\, z + y\, D(x,z), \forall\, x, y, z \in S$

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Definition: 2.6

Let S be a semiprime semiring. A biderivation B and a derivation D of S are called **orthogonal** if B(x,y) S D(z) = 0 = D(z) S B(x,y), $\forall$ x, y, z $\in$ S

Lemma: 2.7

Let S be a 2-torsionfree semiprime semiring and a, b $\in$ S, then the following conditions are equivalent

(i) axb = 0, $\forall$ x $\in$ S
(ii) bxa = 0, $\forall$ x $\in$ S
(iii) axb + bxa = 0, $\forall$ x $\in$ S. If one of the conditions is fulfilled then ab = ba = 0

III. CONDITIONS FOR ORTHOGONALITY

Lemma: 3.1

Le t S be a semiprime semiring. Suppose that an additive mapping h on S and a biaddiive mapping f : S X S $\to$ S satisfy f(x,y) S h(x) = 0, $\forall x, y \in S$. Then f(x,y) S h(z) = 0, $\forall x, y, z \in S$.

Proof:

Given, f(x,y) S h(x) = 0, $\forall x, y \in S$. ---------------------------- (1)

$\Rightarrow$ f(x,y) z h(x) = 0, $\forall x, y, z \in S$.

Linearizing on x gives for all $x, y, z, s \in S$ such that f(x + s,y) z h(x+ s) = 0

$\Rightarrow$ f(x,y) z h(x) + f(s,y) z h(s) + f(x,y) z h(s) + f(s,y) z h(x) = 0

Using (1) we get, f(x,y) z h(s) + f(s,y) z h(x) = 0

Ie, f(x,y) z h(s) = - f(s,y) z h(x) , $\forall x, y, z, s \in S$

Premultiply by f(x,y) z h(s) S on both sides we get,

f(x,y) z h(s) S f(x,y) z h(s) = - f(x,y) z h(s) S f(s,y) z h(x) , $\forall x, y, z, s \in S$

= 0 [by (1)

Since S is semiprime, f(x,y) z h(s) = 0.

Lemma: 3.2

Let S be a 2-torsion free semiprime Semiring. A biderivation B and a derivation D are orthogonal iff B(x,y) D(z) + D(x) B(z,y) = 0, $\forall x, y, z \in S$

Proof:

Suppose B and D such that B(x,y) D(z) + D(x) B(z,y) = 0, $\forall x, y, z \in S$

Replace z by zx, B(x,y) D(zx) + D(x) B(zx,y) = 0, $\forall x, y, z \in S$ ------------------ (2)

$\Rightarrow$ B(x,y) z D(x) + B(x,y) D(z) x + D(x) z B(x,y) + D(x) B(z,y) x = 0, $\forall x, y, z \in S$

$\Rightarrow$ B(x,y) z D(x) + D(x) z B(x,y) + [B(x,y) D(z) + D(x) B(z,y)] x = 0 ------------------- (3)

Using (2) in (3) we get, B(x,y) z D(x) + D(x) z B(x,y) = 0, $\forall x, y, z \in S$ ------------------- (4)

Using lemma 2.7, D(x) S B(x,y) = 0, $\forall x, y, z \in S$

By lemma 3.1, D(x) S B(x,y) = 0, $\forall x, y, z \in S$

Therefore D(x) S B(z,y) = 0, $\forall x, y \in S$ ------------------ (5)

By lemma 2.7, B(x,y) S D(x) = 0 ------------------ (6)

From (5) and (6) we get, B and D are orthogonal.

Conversely, if B and D are orthogonal, then D(x) B(z,y) = B(x,y) D(z) = 0

By lemma 2.7, D(x) B(z,y) + B(x,y) D(z) = 0

Lemma: 3.3

Let D be a derivation and B be a biderivation of a semiring S. The following identity holds, $\forall x, y, z \in S$

(DB) (xy,z) = (DB) (x,z) y + D(x) B(y,z) + B(x,z) D(y) + x (DB) (y,z)

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Theorem: 3.4

Let S be a 2-torsionfree semiprime semiring. A biderivation B and a derivation D are orthogonal iff DB = 0

Proof:

Assume DB = 0.

Using lemma 3.3, (DB) (xy,z) = (DB) (x,z) y + D(x) B(y,z) + B(x,z) D(y) + x (DB) (y,z)

$\Rightarrow$ 0 = D(x) B(y,z) + B(x,z) D(y), $\forall x, y, z \in S$

By lemma 3.2, D and B are orthogonal.

Conversely, if D and B are orthogonal, then D(x) s B(y,z) = 0, $\forall x, y, z, s \in S$ ----------- (7)

Now D[D(x) s B(y,z)] = 0

$\Rightarrow$ D(D(x)) s B(y,z) + D(x) D(s) B(y,z) + D(x) s (DB) (y,z) = 0

Using (7) we get, D(x) s (DB) (y,z) = 0

Let x = B(y,z)

Then D(B(y,z)) s (DB) (y,z) = 0

$\Rightarrow$ (DB) (y,z) S (DB) (y,z) = 0

Since S is semiprime , (DB) (y,z) = 0, $\forall y, z \in S$

Ie, DB = 0

Theorem: 3.5

Let S be a 2-torsion free Semiprime Semiring. A biderivation B and a derivation D are orthogonal iff D(x) B(x,y) = 0 or D(x) B(y,x) = 0, $\forall x, y \in S$

Proof :

Assume D(x) B(x,y) = 0, $\forall x, y \in S$ ------------------ (8)

Linearization on x for (8) gives, D(x + z) B(x + z,y) = 0

$\Rightarrow$ D(x) B(x,y) + D(x) B(z,y) + D(z) B(x,y) + D(z) B(z,y) = 0

$\Rightarrow$ D(x) B(z,y) + D(z) B(x,y) = 0 ------------------ (9)

Take z = zs in (9) gives, D(x) B(zs,y) + D(zs) B(x,y) = 0

$\Rightarrow$ D(x) z B(s,y) + D(x) B(z,y) s + D(z) s B(x,y) + z D(s) B(x,y) = 0, $\forall x, y, z, s \in S$ --------- (10)

(9) $\Rightarrow$ D(x) B(z,y) = - D(z) B(x,y) and D(s) B(x,y) = - D(x) B(s,y)

Substitute these values in (10) we get,

D(x) z B(s,y) – D(z) B(x,y) s + D(z) s B(x,y) – z D(x) B(s,y) = 0 ------------------ (11)

Replacing z by D(x) in (11), D(x) D(x) B(s,y) – D (D(x)) B(x,y) s + D (D(x)) s B(x,y) – D(x) D(x) B(s,y) = 0

$\Rightarrow$ $D^2(x)$ [s,B(x,y)] = 0, $\forall x, y, s \in S$ ------------------ (12)

Put s = sw in last equation we get, $D^2(x)$ [sw,B(x,y)] = 0

$D^2(x)$ s [w,B(x,y)] + $D^2(x)$ [s,B(x,y)] w = 0

Using (12) in the last relation we get, $D^2(x)$ s [w,B(x,y)] = 0

By lemma 3.1, $D^2(x)$ S [w,B(r,y)] = 0, $\forall x, y, w, r \in S$ ------------------- (13)

Replacing x by xu in (13), $D^2(xu)$ S [w,B(r,y)] = 0

($D^2(x)u + 2D(x)D(u) + x\ D^2(u)$) S [w,B(r,y)] = 0

Using (13) we get, 2 D(x) D(u) S [w,B(r,y)] = 0

$\Rightarrow$ D(x) D(u) S [w,B(r,y)] = 0 ------------------- (14)

Taking x = xz in (14) we get D(xz) D(u) S [w,B(r,y)] = 0

D(x) z D(u) S [w,B(r,y)] + x D(z) D(u) S [w,B(r,y)] = 0
$\Rightarrow$ D(x) S D(u) S [w,B(r,y)] = 0
In particular, D(x) S [w,B(r,y)] S D(x) S [w,B(r,y)] = 0
Since S is semiprime semiring, D(x) S [w,B(r,y)] = 0
But , [D(x),B(r,y)] S [D(x),B(r,y)] = 0
Hence D(x) B(r,y) = B(r,y) D(x)
Therefore (9) can be written as B(r,y) D(x) + D(r) B(x,y) = 0
By lemma 3.2, D and B are orthogonal, similarly we can prove that if D(x) B(y,x) = 0, then D and B are orthogonal.
Conversely, if D and B are orthogonal, then D(x) S B(x,y) = 0
Therefore D(x) B(x,y) = 0

Theorem : 3.6

Let S be a 2- torsion free semiprime semiring. Then a biderivaion B and a derivation D are orthogonal iff DB is biderivation

Proof:

Let B and D be such that DB is a biderivation.
Then (DB) (xy,z) = (DB) (x,z) y + x (DB) (y,z), $\forall x, y, z \in S$ --------------- (15)
By lemma 3.3, we get,
(DB) (xy,z) = (DB) (x,z) y + D(x) B(y,z) + B(x,z) D(y) + x (DB) (y,z) --------------- (16)
Substitute (15) in (16) we get, D(x) B(y,z) + B(x,z) D(y) = 0
By lemma 2.3 we get, D and B be orthogonal.
Conversely, let D and B be orthogonal. Then by lemma 3.2 gives, D(x) B(y,z) + B(x,z) D(y) = 0
Now , (DB) (xy,z) = (DB) (x,z) y + D(x) B(y,z) + B(x,z) D(y) + x (DB) (y,z)
$\Rightarrow$ (DB) (xy,z) = (DB) (x,z) y + x (DB) (y,z) [by (16)]
Therefore DB is a biderivation.

Lemma: 3.7

Let S be a 2- torsion free semiprime Semiring and let B : S X S $\to$ S be a Jordan biderivation. Then B is a biderivation

Theorem : 3.8

Let S be a 2- torsion free semiprime Semiring. Then a biderivation B and a derivation D are orthogonal iff D(x) B(x,y) + B(x,y) D(x) = 0, $\forall x, y \in S$

Proof:

Assume D(x) B(x,y) + B(x,y) D(x) = 0, $\forall x, y \in S$
We know that, (DB) (xy,z) = (DB) (x,z) y + D(x) B(y,z) + B(x,z) D(y) + x (DB) (y,z)
Take y = x
Then (DB) (x^2 ,z) = (DB) (x,z) x + D(x) B(x,z) + B(x,z) D(x) + x (DB) (x,z)
= (DB) (x,z) x + x (DB) (x,z)
Therefore DB is a Jordan biderivation. By lemma 3.7, DB is a biderivation, By theorem 3.6, D and B are orthogonal
Conversely, let D and B be orthogonal.
By lemma 3.2, D(x) B(y,z) + B(x,z) D(y) = 0, $\forall x, y, z \in S$.
Put y = x, D(x) B(x,z) + B(x,z) D(x) = 0

Now we prove the main theorem of this section

Theorem : 3.9

Let S be a 2- torsion free semiprime Semiring, B is a biderivation and D is a derivation on S. Then B and D are orthogonal iff the following conditions are equivalent

i) DB = 0

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ii) D(x) B(x,y) = 0 or D(x) B(y,x) = 0, $\forall\, x, y \in S$

iii) DB is a biderivation

iv) D(x) B(x,y) + B(x,y) D(x) = 0, $\forall\, x, y \in S$

Proof:

It follows easily from theorem 3.5, 3.6 and 3.8

Theorem : 3.10

Let S be a 2- torsion free semiprime semiring. Then a biderivation B and a derivation D are orthogonal, if there exists a $\in$ S such that (DB) (x,y) = xay + yax, $\forall\, x,\ y \in S$

Proof:

Let a be a fixed element in S, B is a biderivation and D is a derivation satisfying

(DB) (x,y) = xay + yax, $\forall\, x, y \in S$ --------------- (17)

Consider, (DB) (xy,z) = (DB) (x,z) y + D(x) B(y,z) + B(x,z) D(y) + x (DB) (y,z) ----- (18)

Substitute (17) in (18),

(DB) (xy,z) = (xaz + zax) y + D(x) B(y,z) + B(x,z) D(y) + x (yaz + zay)

= xazy + zaxy + xyaz + xzay + D(x) B(y,z) + B(x,z) D(y) --------------- (19)

Now (DB) (xy,z) = xyaz + zaxy --------------- (20)

Comparing (19) & (20) we get,

xyaz + zaxy = xazy + zaxy + xyaz + xzay + D(x) B(y,z) + B(x,z) D(y)

0 = x(za + az) y + D(x) B(y,z) + B(x,z) D(y) -------------- (21)

Replacing y by yx in (21), x (za +az) yx + D(x) B(yx,z) + B(x,z) D(yx) = 0

$\Rightarrow$ x (za + az) yx + D(x) y B(x,z) + D(x) B(y,z) x + B(x,z) y D(x) + B(x,z) D(y) x = 0

$\Rightarrow$ [x (za + az) y + D(x) B(y,z) + B(x,z) D(y)] x + D(x) y B(x,z) + B(x,z) y D(x) = 0

$\Rightarrow$ D(x) y B(x,z) + B(x,z) y D(x) = 0 [by (21)]

By lemma 2.7, D(x) S B(x,z) = 0,

By lemma 3.1, D(x) S B(y,z) = 0

Again by lemma 2.7, B(y,z) S D(x) = 0

Therefore D and B are orthogonal.

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